STAT 291

Problems using the Gamma or Beta Distributions

Some strains of paramecia secrete "killer" particles that will cause the death of a sensitive individual if contact is made. All paramecia unable to produce killer particles are sensitive. The mean number of killer particles emitted by a killer paramecium is 1 every 5 hours.

1. In observing such paramecium, what is the probability that we must wait at most 4 hours before the first killer particle is emitted?

2. What is the probability that we would have to wait more than 6 hours?

3. What is the mean wait time? The standard deviation of wait times?

4. Given that we have waited four hours, what is that probability that we will have to wait a total of 8 hours?

In a certain city, the daily consumption of electric power (in millions of kilowatt-hours) can be treated as a random variable having a gamma distribution with $\alpha = 3$ and $\beta = 2$.

1. What are the mean and standard deviation of this distribution?

2. Write down the density function for this setting, being sure to simplify as much as reasonable.

3. If we know that the daily capacity of the power plant for the city is 12 million kilowatts-hours, what is the probability that the city usage will exceed this capacity?

4. What is the probability that between 5 and 10 million kilowatt-hours will be used on a random day?

Ceramic parts are manufactured in large batches. Because of production hazards, the fraction X of a batch which is saleable is a random variable. It is known, from past experience, that X has a standard beta distribution with $\alpha = 6$ and $\beta = 3$.

1. Write down the density function for this setting, simplifying as much as possible.

2. Find F(x) for this setting.

3. What is the probability that less than half of a randomly chosen batch is saleable?

4. What is the probability that more than 75% of a batch is saleable?

5. What are the mean and standard deviation for X?