NAME ______________________________

STAT 291 - Statistics for the Mathematical Sciences I

Hypothesis Testing

1. **Hypotheses:** The first step is to clearly state the hypotheses that we are testing in terms of the parameters involved, and to clearly define the parameters in terms of the problem at hand. There will always be two hypotheses, the null and the alternative. For our purposes, the alternative and the null hypotheses will always be the complements.

   - The Null Hypothesis will always contain a statement of equality. Generally this is the "status quo" hypothesis, that nothing has changed from some standard, or that the subset of interest has the same characteristic as the larger population.
   - The Alternative Hypothesis is often called the "research hypothesis" and is generally a statement of what we believe, or hope, to be true.

2. **Assumptions:** Each method we discuss will make certain assumptions about the way we collected our data and the nature of the population, or populations, from which they came. Stating these assumptions with each test will remind us what we are assuming, and that these assumptions can affect the validity of the test. If any of the assumptions are not valid, the conclusions we reach may not be trustworthy.

3. **Rejection Region:** In this step we will determine what values of the Test Statistic (see the next step) will lead us to reject the null hypothesis in favor of the alternative, and which values will not enable us to reject the null hypothesis.

4. **Test Statistic:** This is a statistic (based on our data) whose distribution is known under the assumption that the null hypothesis is true. If the value is abnormal for this distribution, it causes us to doubt, and possibly reject, the null hypothesis.

5. P**-value:** This is a measure of the probability, assuming the null is true, that we would see a test statistic as unusual, or "rare", as the one we observed.

6. **Conclusion:** We will then compare the Test Statistic to our Rejection Region, and our P-value to our predetermined cut off value α and determine our conclusion to either reject the null hypothesis or fail to reject the null hypothesis. Note that we will never "accept the null hypothesis". This conclusion should be stated clearly in terms of the problem at hand, in plain English. "Reject the null" or "Fail to reject the null" is not sufficient.

Hypothesis Testing Details: For μ, large sample

1. **Hypotheses:**

   H_0 $\mu = \mu_0$ $\quad$ $\mu \leq \mu_0$ $\quad$ $\mu \geq \mu_0$

   H_a $\mu \neq \mu_0$ $\quad$ $\mu > \mu_0$ $\quad$ $\mu < \mu_0$

   Where: μ is the mean ____________ for all ____________________

2. **Assumptions:** There are actually two options here:

   We have independent, random observations from some population, and the sample size is large enough that we can use the Central Limit Theorem.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < Z_\alpha$

   Right: Reject H_0 if $TS > Z_{1-\alpha}$

   Two: Reject H_0 if $TS < Z_{\alpha/2}$ or if $TS > Z_{1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

5. **P-value:** For the three types of tests:

   Left: $P - \text{value} = P(Z < TS)$

   Right: $P - \text{value} = P(Z > TS)$

   Two: $P - \text{value} = 2 \cdot P(Z < - \mid TS \mid)$

6. **Conclusion:** We (do not) have enough evidence to conclude that the mean ____________ for all ____________________ is (more than/less than/not) (value of μ_0). (If two tailed and we reject H_0, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For μ, small sample

1. **Hypotheses:**

   H_0 $\mu = \mu_0$ $\quad \mu \leq \mu_0 \quad \mu \geq \mu_0$

   H_a $\mu \neq \mu_0$ $\quad \mu > \mu_0 \quad \mu < \mu_0$

   Where: μ is the mean ____________ for all __________________

2. **Assumptions:** We have independent, random observations from a normally distributed population, with unknown variance.

3. **Rejection Region:** For the three types of tests:

   Left: Reject H_0 if $TS < t_{n-1,\alpha}$

   Right: Reject H_0 if $TS > t_{n-1,1-\alpha}$

   Two: Reject H_0 if $TS < t_{n-1,\alpha/2}$ or if $TS > t_{n-1,1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$$

5. **P-value:** For the three types of tests:

   Left: $P - \text{value} = P(t_{n-1} < TS)$

   Right: $P - \text{value} = P(t_{n-1} > TS)$

   Two: $P - \text{value} = 2 \cdot P(t_{n-1} < - \mid TS \mid)$

6. **Conclusion:** We (do not) have enough evidence to conclude that the mean ____________ for all __________________ is (more than/less than/not) (value of μ_0). (If two tailed, add: "In fact, it is (more/less).")

Hypothesis Testing Details: For p

1. **Hypotheses:**

   H_0 $p = p_0$ $\quad p \leq p_0 \quad p \geq p_0$

   H_a $p \neq p_0$ $\quad p > p_0 \quad p < p_0$

   Where: p is the true proportion of all ____________ that __________________

2. **Assumptions:** We have independent, random observations from a binomial experiment, and there are enough trials that we can use the Central Limit Theorem.

3. **Rejection Region:** For the three types of tests:

Left: Reject H_0 if $TS < Z_\alpha$

Right: Reject H_0 if $TS > Z_{1-\alpha}$

Two: Reject H_0 if $TS < Z_{\alpha/2}$ or if $TS > Z_{1-\alpha/2}$

4. **Test Statistic:**

$$TS = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot (1-p_0)}{n}}}$$

5. **P-value:** For the three types of tests:

Left: $P - \text{value} = P(Z < TS)$

Right: $P - \text{value} = P(Z > TS)$

Two: $P - \text{value} = 2 \cdot P(Z < - \mid TS \mid)$

6. **Conclusion:** We (do not) have enough evidence to conclude that the proportion of all ____________ that __________________ is (more than/less than/not) (value of p_0). (If two tailed, add: "In fact, it is (more/less).")